

DOCUMENT RESUME

ED 206 641

TM 810 529

AUTHOR Huberty, Carl J.; Smith, Jerry D.
 TITLE The Study of Effects in MANOVA.
 PUB DATE Apr 81
 NOTE 24p.; Paper presented at the Annual Meeting of the American Educational Research Association (65th, Los Angeles, CA, April 13-17, 1981).

EDRS PRICE MF01/PC01 Plus Postage.
 DESCRIPTORS *Analysis of Variance; Comparative Analysis; Discriminant Analysis; Hypothesis Testing; *Multivariate Analysis
 IDENTIFIERS *Contrasting Groups Method; *Linear Discriminant Function

ABSTRACT

A particular strategy for investigating effects resulting from a multivariate analysis of variance (MANOVA) is proposed. The strategy involves multiple two-group multivariate analyses. The two groups result from considering multivariate pairwise group contrasts or multivariate complex group contrasts. Assuming a given two-group analysis yields real effects, the resultant single linear discriminant function (LDF) is studied. A transformation of LDF weights (Urbakh) is recommended for assessing variable relative contribution. The analysis strategy is described in detail and illustrated with real data sets. (Author/BW)

 * Reproductions supplied by EDRS are the best that can be made *
 * from the original document. *

ED206641

U.S. DEPARTMENT OF EDUCATION
NATIONAL INSTITUTE OF EDUCATION
EDUCATIONAL RESOURCES INFORMATION
CENTER (ERIC)

* This document has been reproduced as
received from the person or organization
originating it.

Minor changes have been made to improve
reproduction quality.

* Points of view or opinions stated in this docu-
ment do not necessarily represent official NIE
position or policy.

The Study of Effects in MANOVA

Carl J Huberty Jerry D. Smith
University of Georgia

"PERMISSION TO REPRODUCE THIS
MATERIAL HAS BEEN GRANTED BY

J. D. Smith

TO THE EDUCATIONAL RESOURCES
INFORMATION CENTER (ERIC)."

Paper presented at the annual meeting of the American Educational
Research Association, Los Angeles, April 1981.

TM 810529

Abstract

A particular strategy for investigating effects resulting from a MANOVA is proposed. The strategy involves multiple two-group multivariate analyses. The two groups result from considering multivariate pairwise group contrasts or multivariate complex group contrasts. Assuming a given two-group analysis yields real effects, the resultant single linear discriminant function (LDF) is studied. A transformation of LDF weights, due to V.Y. Urbakh, is recommended for assessing variable relative contribution. The analysis strategy is described in detail and illustrated with real data sets.

The Study of Effects in MANOVA

Consider two research situations. First, suppose a researcher is interested in studying the joint and separate effects of (1) differing goal structures, and (2) differing age mixtures of triads of children on a collection of measures pertaining to cooperative triad performance. Second, suppose one is interested in describing students in three different types of post high school education settings with respect to a collection of achievement, interest, temperament, and socioeconomic measures. An "effect" may be thought of as an antecedent-consequent result in an experimental setting, such as the effect of goal structure on cooperative performance. Or, an effect may be thought of as a static sample mean deviation from a grand mean that would be of interest when describing group differences in a "natural" setting, such as three samples of post secondary youths.

When a researcher's study involves two or more samples of experimental units, for each unit on which there are p response variable measures, a statistical technique of interest is multivariate analysis of variance (MANOVA). This may be the case when there is more than one grouping variable (as in the first situation above) or when there is only one grouping variable (as in the second situation above). MANOVA is appropriate when certain data conditions are tenable. These conditions are direct extensions of those requisite for univariate analysis of variance (ANOVA): independence of the observation vectors; population (corresponding to each sample) multivariate normality of the vectors; and equality of the vector population covariance matrices.

In this paper we are dealing with the research situation in which the MANOVA requisite data conditions listed in the above paragraph are considered tenable. In particular, we want to discuss an analysis strategy and procedure which a researcher can employ to interpret MANOVA effects. Values of proposed statistics to be discussed are readily accessible through widely used computer program packages, SAS and BMDP. Discussion of the proposed strategy is limited to a situation involving at most two grouping variables; i.e., to a two-way or a one-way MANOVA layout.

The proposed strategy is basically the same strategy as that often employed in an ANOVA situation, a study of group contrasts. For example, it may be of interest to contrast Goal Structure Condition 1 with Condition 2; or, contrast university freshmen with business and vocational college freshmen combined.

There are two purposes of this paper. One is to describe an analysis strategy in the study of MANOVA effects. The second purpose is to explicitly describe and illustrate how the analysis may be accomplished.

Assessing Contrast Effects

In assessing multivariate group contrasts one may take either of two approaches: (1) perform the omnibus test(s) first, followed by a look at some contrasts, or (2) proceed directly to group contrasts in answering specific planned or a priori research questions. The second approach is particularly appropriate when there is one grouping variable having more than two levels. [If group contrasts are not of interest at all, then the usual descriptive discriminant analysis techniques are appropriate (see Tatsuoaka, 1973, pp. 280, 300-302).] The specified contrasts may be either of the pairwise or complex variety. An example of a pairwise

contrast is

$$\psi_1 = \underline{\mu}_1 - \underline{\mu}_3,$$

where $\underline{\mu}_g$ is a $(p \times 1)$ vector of means on the p response variables for population g . An example of a complex contrast is

$$\psi_2 = (\underline{\mu}_1 + \underline{\mu}_3) - (\underline{\mu}_2 + \underline{\mu}_4).$$

Note that such contrasts, pairwise or complex, involve contrasting mean vectors; that is, contrasting groups over the p response variables simultaneously.

If desired, tests of the omnibus hypothesis in a one-way MANOVA context or the three omnibus hypotheses in a two-way context may be accomplished using either SAS's GLM program, or BMDP7M. [With the latter for a two-way layout, a contrast matrix must be specified for each of the three hypotheses.] A popular MANOVA criterion is that due to S.S. Wilks, the so-called lambda criterion, Λ . A transformation of Λ , due to C.R. Rao, leads to an approximate F statistic. One index that can be used to assess effects reflected by the omnibus hypothesis is the P value associated with the value of the F statistic. A second index of the presence of effects is $\eta^2 = 1 - \Lambda$. The η^2 , like that in a univariate context, is a variance-explained index. For a given test, if the P value is "small" and the η^2 value is "respectable," then it may be concluded that real effects are present.

The contrast analyses may be accomplished using the BMDP7M program, Stepwise Discriminant Analysis. [It should be noted that our purpose in using BMDP7M is to study contrast effects, not to use the stepwise aspect of the program to "select" variables. It is assumed that the researcher has judiciously chosen variables for analysis and, possibly, screened out some variables prior to the MANOVA.] A few special BMDP control paragraphs and statements need to be noted. In the DISCRIMINANT paragraph,

the F-to-enter limit (ENTER) and F-to-remove limit (REMOVE) should be set low, low enough to get all of the variables of interest into the analysis. In the same paragraph, the CONTRAST statement reflecting the contrast of interest is included. [So that the contrast printed by the program is the same as that in the CONTRAST statement, integers should be used in the statement.] Finally, to get the eigenanalysis for the contrast specified, it is necessary to include the statement CONTRAST in the PLOT paragraph.

Just as in assessment of omnibus MANOVA effects, two indices of contrast effects may be considered. One index is the P value associated with the value of the F statistic, a transformation of Λ . The second index is $\eta^2 = 1 - \Lambda$.

Variable Contribution

Assume, for a given contrast, there is a real effect present. If so, interpretation proceeds to the resulting single linear discriminant function (LDF) yielded by BMDP7M. A function of the LDF weights may be examined to assess the relative contribution of the variables to the group separation reflected by the contrast of interest. The suggested variable contribution index is due to Urbakh (1971):

$$u_1 = (b_1^*)^2 / r^{11},$$

where b_1^* is the "standardized LDF weight" for variable 1, and r^{11} is the 11th diagonal element of the inverse of the (pxp) within-cells response variable correlation matrix, R. The Urbakh index may also be determined using $s_{11} b_1^2 / r^{11}$, where s_{11} is the 11th diagonal element of the within-cells covariance matrix, S, and b_1 is the 11th LDF weight applicable to an unstandardized response variable score.

One justification for the use of this index is that u_1 is an estimate of the decrease in group separation (i.e., decrease in the Mahalanobis distance index) when variable 1 is deleted (see Urbakh, 1971, p. 532). A second justification is that by using this index, collinearity between variable 1 and the other $p-1$ variables is "taken into consideration." This is true because the multiple correlation, $R_{(1)}$, between variable 1 on the one hand and the remaining $p-1$ variables on the other hand reflects the degree of collinearity for variable 1, and (see Kerlinger & Pedhazur, 1973, p. 67)

$$r^{11} = [1 - R_{(1)}^2]^{-1}.$$

[The r^{11} value is sometimes called a "variance inflation factor" (see Freund & Minton, 1979, p. 119; and Willan & Watts, 1978).]

It may be noted that

$$R = D^{-1/2} S D^{-1/2},$$

where S is the within-cells covariance matrix and $D = \text{diag}(S)$. The matrices S , D , R , and R^{-1} are readily obtainable using SAS's PROC MATRIX.

Thus, it is possible to determine the variables that contribute most (and least) to the resultant effect reflected by a given contrast. It may be informative to discover that some variables contribute to some effects and not to others, or that some variables contribute to many effects.

Summary

The procedure discussed in the last two sections of this paper for carrying out the complete analysis is summarized in Figure 1. The SPSS

Insert Figure 1 about here

DISCRIMINANT program (with Option 15) may also be used to obtain a punched S

matrix. It should be noted that for a two-way layout, "contrasts" are used with BMDP7M in the analysis in two phases. First, they are used to accomplish the omnibus analyses. Next, they are used to study specific comparisons among the cells/groups. It is the latter contrasts that are of specific interest in this paper.

Illustrative Examples

Data Set A¹

These data are based on triads of young children; that is, the unit of analysis is a triad. Two different types of triads were considered: one with two first graders and one third grader, and another with three third graders. This gives an "Age Mixture" dimension with two levels. Triads of children were randomly assigned to four different Goal Structure conditions. There were 19 triads for each condition for the first age level, and 18 triads for each condition for the second age level. Six response measures were obtained on each of the 148 triads. These were measures of triad cooperative performance, and pertained to a tower-building task. So, here we have a two-way, six-variate analysis of variance situation with proportional cell frequencies:

		Condition			
		C ₁	C ₂	C ₃	C ₄
Age	A ₁	19	19	19	19
	A ₂	18	18	18	18

We are assuming that the requisite data conditions for MANOVA are tenable. The BMDP7M program, with a contrast coefficient matrix was used for the omnibus analyses. The test for the six-variate Age-by-Condition interaction effects yielded nonsignificance ($P > .50$). Tests for Age and Condition main effects both yielded significance ($P < .005$, $\eta^2_{\text{Age}} = .13$, and

$P < .03, \eta^2_{\text{cond}} = .21$).

The Condition main effects are judged to be real. Thus, to illustrate the proposed strategy, contrasts among Conditions will be considered. Seven contrasts were specified; the six pairwise contrasts, plus the contrast of Conditions 1 and 3 versus Conditions 2 and 4. The coefficients for these contrasts, used with BMDP7M, are given in Table 1.²

Insert Table 1 about here

[A table of means, univariate Fs, and eigenvalues for Condition effects, including the within-cell correlations, all for Data Set A, is available upon request.]

Two of the six pairwise contrasts led to clear nonsignificance: Condition 2 versus Condition 3, Rao's F value is $F = 1.36$, the P value is $> .20$; Condition 2 versus Condition 4, $F = 0.40$, $P > .80$. Two other pairwise contrasts yielded apparent real effects: Condition 1 versus Condition 2, $F = 3.04$, $P < .01$, $\eta^2 = .12$; Condition 1 versus Condition 4, $F = 3.24$, $P < .01$, $\eta^2 = .13$. The remaining two pairwise contrasts yielded somewhat "marginal" results. Condition 1 versus Condition 3, $F = 1.53$, $.10 < P < .20$, $\eta^2 = .06$; Condition 3 versus Condition 4, $F = 1.83$, $.10 < P < .20$, $\eta^2 = .06$. It was judged that the complex contrast yielded real effects: $F = 3.60$, $P < .005$, $\eta^2 = .14$. Therefore, the first two contrasts mentioned in this paragraph were excluded from further discussion. The two marginal contrasts were retained simply for the purpose of illustration of our proposed strategy.

The Urbakh index, u , was used to order the six variables with respect to their relative contribution to separation reflected by each of the five

contrasts. SAS's PROC MATRIX was used to obtain the u values; the BMDP7M printed LDF weights were used as partial input. The five sets of standardized LDF weights, the five sets of u values, and the five variable rankings are presented in Table 2.

Insert Table 2 about here

It may be observed from Table 2 that variables contribute in a somewhat differential manner to group separation as reflected by the five contrasts. Even though the agreement among the five sets of rankings may be considered statistically significant,³ some disparity exists. For example, Variable 2 is essentially a noncontributor to separation between Conditions 1 and 2, Conditions 1 and 3, and Conditions 1 and 4, but it is the biggest contributor to the separation between Conditions 3 and 4. Also, Variable 6 contributes most to separation between Conditions 1 and 3, but relatively little to other separations.

Supposing, from a substantive viewpoint, that the contrasts specified here were those of most import, an overall ordering of variable contribution is obtainable by ordering the rank sums. The overall ordering of the variables is: 1, 2, 4, 3, 6, 5.

Substantive interpretation of the results for this data set is beyond the scope of this paper. When considering variable and Condition definitions, however, the above type of information should contribute greatly to substantive interpretation. Relative variable contribution interpretation based on multiple ANOVA results, on standardized multiple-group linear discriminant function weights, or on stepwise discriminant analysis are less defensible.

Data Set B⁴

Fifteen measures on each subject in three post high school progress groups comprise the second set of data. Three samples were randomly selected from a nationwide stratified sample of nearly 26,000 eleventh grade students. The sampling, stratified with respect to sex, produced a total of 200 students. Measures on the 15 variables were obtained in 1960, while group membership was determined in 1962. Six of the measures are in the cognitive domain; five are interest measures; three are temperament measures; the final measure is that of composite socioeconomic status. Group 1 consisted of 26 students who enrolled in a business college. Group 2 consisted of 75 vocational college students. Group 3 consisted of 99 university students.

The tenability of the requisite data conditions for MANOVA was assumed. The three-group MANOVA yielded $\Lambda = .544$ ($F = 4.34^5$, $P < .001$, $\eta^2 = .46$). Four contrasts were specified; the three pairwise contrasts, plus the contrast of Groups 1 and 2 versus Group 3.

[A table of the Group means, univariate Fs, within-groups correlations, and eigenvalues for Data Set B is available upon request.]

For the sake of this discussion, it was judged that all four contrasts yielded "real" effects: Group 1 versus Group 2, $F = 1.60$, $P < .05$, $\eta^2 = .12$; Group 1 versus Group 3, $F = 4.07$, $P < .001$, $\eta^2 = .25$; Group 2 versus Group 3, $F = 6.75$, $P < .001$, $\eta^2 = .36$; and Groups 1 and 2 versus Group 3, $F = 6.82$, $P < .001$, $\eta^2 = .36$.

The four sets of u values are given in Table 3, along with the four sets of standardized LDF weights. There appears to be a fair bit of agreement

Insert Table 3 about here

among the four sets of rankings.⁶ The "best" variable for differentiating Group 1 (business college students) from Group 2 (vocational college students) is Variable 11 (interest in skilled trades). The variables contributing most to the separation between Group 1 and Group 3 (university students) are Variable 5 (mathematics information), Variable 7 (interest in physical science), Variable 14 (mature personality), and Variable 3 (proficiency in English). Separation between Groups 2 and 3 is most attributable to Variable 11. Variable 5 ranks best in terms of the separation of Groups 1 and 2 from Group 3.

Considering, from a substantive viewpoint, that these four contrasts as those of basic interest, then an overall variable ordering is obtainable. The rank sums (over the four sets of ranks) yield the following variable ordering: 11, 5, 7, 14, 3 and 8, 2, 6, 10, 1, 4, 9, 15, 12, 13.

Comments

The focus of this paper is on the use of multivariate group contrasts. An analysis strategy is proposed that can be expected to provide information helpful in the interpretation of grouping variable effects reflected by group contrasts in a multivariate analysis of variance (MANOVA) context. The context may be that of a one- or higher-way layout. The Urbakh (1971) index is suggested for use in assessing response variable contribution to resultant effects. An ordering of the variables may be obtained with respect to each contrast specified and over all contrasts.

There are at least three advantages to the proposed strategy. First, the strategy encourages the researcher to think about effects of interest, and to relate these to some underlying theory or to beneficial decisions/outcomes of the research. Asking questions about specific multivariate group contrasts should yield more precise information than asking omnibus

questions across all groups involved in a study. Furthermore, and perhaps more importantly, the strategy encourages the researcher to think multivariately; i.e., not to ignore response variable intercorrelations. [Another aspect of "multivariate thinking" will be discussed below.]

The second advantage of our strategy pertains to a very specific meaning of variable contribution. Urbakh (1971, p. 532) showed that his index is an estimate of the decrease in group separation when a given variable is omitted from the analysis. Thus, the researcher gets a clear idea of the relative importance of the variables in terms of their contribution to the separation reflected by each contrast investigated. The researcher has an idea of which variables are most important for what separation. This clarity is not present for the researcher who examines weights of the standardized linear discriminant functions (LDFs) resulting from a multiple-group omnibus analysis.

The third advantage is one that appears to overcome another shortcoming associated with variable orderings based on standardized multiple-group LDF weights alone. It is well known that when the response variables are highly intercorrelated (the multicollinearity problem), the LDF weights lack stability over repeated sampling. That is, a variable ordering based on LDF weights for a given sample may be quite different from an LDF weight ordering for other samples. It is conjectured that the Urbakh index yields a more reliable ordering, since the square of the two-group LDF weight is divided by a measure of collinearity for the given variable. The reliability contention needs to be studied further, of course.

A word of caution is in order. The variable ordering yielded by the Urbakh index is appropriate only when the complete set of variables is considered. For a set of p variables, u_1 is an estimator for $\Delta_p^2 - \Delta_{p-1}^2$,

where Δ_{p-1}^2 is the population Mahalanobis distance index based on all p variables except variable i . If the complete set of variables is not retained for interpretation considerations, then the referent $\hat{\Delta}_p^2$ is different, and the ordering of the variables in the retained subset may be unlike the subset ordering in the company of the discarded variables. The concept of variable ordering discussed in this paper must be interpreted in this somewhat restrictive sense.

Further interpretation of resulting contrast effects may be made by characterizing or identifying the construct underlying the group differences. An effect construct may be characterized or identified by examining the correlations between the single LDF and each variable.⁷ [This is the other aspect of "multivariate thinking" referred to earlier in this section.] These correlations are sometimes called "structural loadings," as in factor analysis. A subset of variables with "high" loadings would be considered in terms of the subset commonality in identifying a construct. It may very well be that different constructs are associated with different contrast effects.

A final comment is made regarding the design and analysis for a two-way (or higher dimension) layout. It may be noted that the two-way example used in this paper has proportional cell frequencies. [Five (out of 153) cases were deleted from the original design to achieve proportionality.] The reason that an orthogonal design was employed is simplicity. There are theoretical, philosophical, and practical issues in the analysis of non-orthogonal designs which are beyond the scope of this paper. [See Blair & Higgins, 1978; Cramer & Appelbaum, 1980; and Herr & Gaebelin, 1978, for discussions and references for the univariate problem.] We subscribe to the position of testing row (column) effects in the presence of column (row)

effects while ignoring interaction effects. Taking such a position, it is easy enough to accomplish the omnibus analyses for a nonorthogonal design (by using, for example, SAS's GLM program with Type II sums of squares).⁸ The problem, however, comes when formulating the "adjusted" means (mean vectors) to be used in contrasts and calculating the standard errors (error covariance matrices) of these adjusted means (mean vectors). It is not appropriate to consider simple weighted or unweighted cell means as the "marginal" means. And, it turns out, the error variances and covariances for the adjusted mean vector differences are complicated functions of elements of a "basis matrix" (see Bock, 1975, pp. 297-300).

References

- Blair, R.C., & Higgins, J. J. Tests of hypotheses for unbalanced factorial designs under various regression/coding method combinations. Educational and Psychological Measurement, 1978, 38, 621-631.
- Bock, R. D. Multivariate statistical methods in behavioral research. New York: McGraw-Hill, 1975.
- Cramer, E. M., & Appelbaum, M. I. Nonorthogonal analysis of variance-once gain. Psychological Bulletin, 1980, 87, 51-57.
- Freund, R. J., & Minton, P. D. Regression methods. New York: Dekker, 1979.
- Herr, D. G., & Gaebalein, J. Nonorthogonal two-way analysis of variance. Psychological Bulletin, 1978, 85, 207-216.
- Kerlinger, F. N., & Pedhazur, E. J. Multiple regression in behavioral research. New York: Holt, Rinehart and Winston, 1973.
- Tatsuoka, M. M. Multivariate analysis in educational research. In F. N. Kerlinger (Ed.), Review of research in education. Itasca, Ill.: Peacock, 1973.
- Urbakh, V. Y. Linear discriminant analysis: Loss of discriminating power when a variate is omitted. Biometrics, 1971, 27, 531-534.
- Willan, A. R., & Watts, D. G. Meaningful multicollinearity measures. Technometrics, 1978, 20, 407-412.

Footnotes

1. The data here are real. The experimental design was modified a little to preserve anonymity.
2. It should be noted that if only pairwise contrasts are of interest, SPSS's DISCRIMINANT may be used exclusively. Option 15 yields punched cell means as well as a punched S matrix. For pairwise contrasts, then, the (pxp) S matrix plus a (px2) mean matrix is used as input for DISCRIMINANT with Option 16. The resulting eigenvalue and Wilks lambda are identical to those obtained from BMDP7M. Also, the standardized LDF weights are printed. A caution to be noted is that the printed Rao F value is incorrect, since the df value used for S is $n_g + n_g - 2$ rather than $N - k$, where k = number of cells and $N = \sum_{g=1}^k n_g$.
3. Agreement among rankings may be assessed using Kendall's coefficient of concordance, r_w . For these five sets of rankings, $r_w = .488$, $.01 < P < .05$.
4. See footnote 1.
5. For three groups, $F = (1 - \sqrt{\Lambda}) (df_e - p + 2) / p \sqrt{\Lambda}$ is an exact F statistic with $v_1 = 2p$ and $v_2 = 2(df_e - p + 2)$.
6. The value of r_w for the rankings in Table 3 is 0.521 ($P < .01$).
7. The LDF-variable correlations may be obtained via SAS's PROC MATRIX, using the expression

$$\underline{r} = \frac{1}{s} D^{-1/2} S \underline{b},$$

where $\underline{r}$ is the (px1) vector of correlations, s is the standard deviation of the LDF, S is the (pxp) within-cells covariance matrix, $D =$

diag(S), and b is the (px1) vector of LDF weights. [A listing of the PROC MATRIX statements to obtain r is available upon request.]

8. If the BMDP7M program is used (with contrast coefficient matrices) for the omnibus nonorthogonal analyses, the row (column) effects are tested in the presence of column (row) and interaction effects.

Table 1

Contrast Coefficients for Data Set A

Condition Contrast	Cell							
	$A_1 C_1$	$A_1 C_2$	$A_1 C_3$	$A_1 C_4$	$A_2 C_1$	$A_2 C_2$	$A_2 C_3$	$A_2 C_4$
1 vs 2	1	-1	0	0	1	-1	0	0
1 vs 3	1	0	-1	0	1	0	-1	0
1 vs 4	1	0	0	-1	1	0	0	-1
2 vs 3	0	1	-1	0	0	1	-1	0
2 vs 4	0	1	0	-1	0	1	0	-1
3 vs 4	0	0	1	-1	0	0	1	-1
1 & 3 vs 2 & 4	1	-1	1	-1	1	-1	1	-1

Table 2
Standardized LDF Weights, Urbakh Index Values,
and Variable Orderings for Contrasts for
Data Set A

Variable	<u>Contrast</u>			<u>Contrast</u>			<u>Contrast</u>		
	1 vs 2			1 vs 3			1 vs 4		
	b_1^*	u_1	Rank	b_1^*	u_1	Rank	b_1^*	u_1	Rank
1	1.06	.42	1	.66	.16	2	1.02	.38	1
2	.49	.09	3	.40	.06	4	.38	.05	4
3	.07	.00	5	.32	.09	3	.14	.02	5
4	.40	.15	2	.11	.01	5	.31	.09	3
5	.22	.03	4	.04	.00	6	.07	.00	6
6	.07	.00	6	.62	.23	1	.43	.11	2

Variable	<u>Contrast</u>			<u>Contrast</u>		
	3 vs 4			1 & 3 vs 2 & 4		
	b_1^*	u_1	Rank	b_1^*	u_1	Rank
1	.82	.25	3	1.07	.42	1
2	.96	.39	1	.76	.21	2
3	.53	.26	2	.20	.03	4
4	.34	.11	4	.42	.16	3
5	.14	.02	5	.21	.03	5
6	.00	.00	6	.04	.00	6

Note: $u_1 = (b_1^*)^2 / r^{11}$; $r^{11} = 2.72$, $r^{22} = 2.78$, $r^{33} = 1.13$, $r^{44} = 1.11$,
 $r^{55} = 1.53$, $r^{66} = 1.65$; b_1^* and u_1 values are rounded to two places;
 absolute b_1^* values are given.

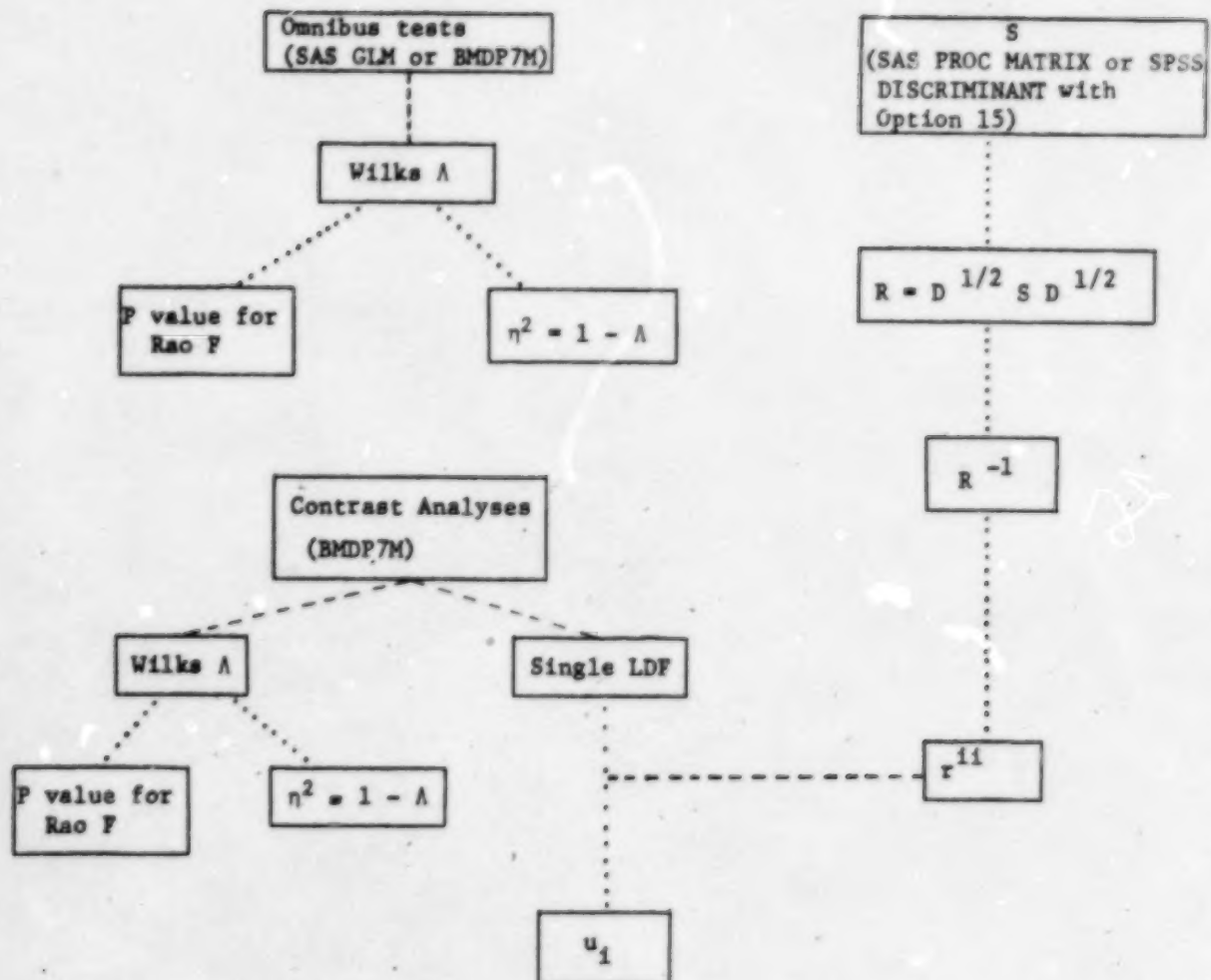
Table 3

Standardized LDF Weights, Urbakh Index Values, and Variable Orderings for Contrasts for Data Set B

Variable	<u>Contrast</u>											
	1 vs 2			1 vs 3			2 vs 3			1 & 2 vs 3		
	b_1^*	u_1	Rank	b_1^*	u_1	Rank	b_1^*	u_1	Rank	b_1^*	u_1	Rank
1	.33	.05	7	.12	.01	13	.37	.06	6	.26	.03	11
2	.05	.00	14	.38	.08	5	.39	.08	4	.41	.09	5
3	.40	.08	2	.43	.10	4	.19	.02	11	.34	.06	6
4	.32	.06	3	.05	.00	15	.28	.05	9	.17	.02	12
5	.14	.01	13	.60	.15	1	.57	.14	2	.62	.16	1
6	.26	.05	5	.32	.07	6	.16	.02	10	.26	.05	8
7	.30	.05	6	.48	.13	2	.32	.06	7	.43	.10	4
8	.32	.06	4	.16	.02	9	.41	.10	3	.30	.05	7
9	.20	.02	12	.17	.01	11	.34	.05	8	.26	.03	10
10	.27	.04	8	.33	.06	7	.17	.02	12	.27	.04	9
11	.74	.31	1	.16	.01	10	.72	.29	1	.45	.11	2
12	.03	.00	15	.12	.01	12	.11	.01	13	.12	.01	13
13	.15	.02	11	.07	.00	14	.03	.00	15	.02	.00	15
14	.16	.02	10	.38	.12	3	.32	.08	5	.37	.11	3
15	.18	.03	9	.16	.02	8	.05	.00	14	.11	.01	14

Note: b_1^* and u_1 values are rounded to two places; ranks are based on unrounded u_1 values; absolute b_1^* values are given.

Figure 1
Analysis Procedure



Output -----

Transformation